

1. Basic Arithmetic and Algebra

It is expected that this topic will receive continual attention over the two years, particularly through applications in other topics. It is left to the teacher's discretion whether to begin treatment of this entire topic immediately or to treat sections as the need arises in conjunction with other topics.

A distinction should be made between exact answers such as $\frac{3}{7}$ or $\sqrt{2}$, and approximate answers which may be obtained when using tables or a calculator.

1.1 The following are included in this section.

- (i) Addition, subtraction, multiplication and division of fractions and decimals.
- (ii) Conversion of fractions (rational numbers) to decimals and percentages, and vice versa. Students should be aware that a fraction (rational number) can be expressed as a terminating or repeating decimal and that conversely such decimals represent rational numbers. Repeating decimals may be converted to fractions by the method of removing the first period.

Example: express $0.\dot{1}\dot{2}$ as a fraction.

$$\begin{aligned}\text{Let } x &= 0.\dot{1}\dot{2}, \\ \text{then } 100x &= 12.\dot{1}\dot{2} \\ &= 12 + x, \\ \text{so } x &= \frac{12}{99} \\ &= \frac{4}{33}.\end{aligned}$$

Generally, if $x = \dot{r}$ is a recurring decimal with period length P , then

$$\begin{aligned}10^P x &= r + \dot{r} \\ \text{so } (10^P - 1)x &= r.\end{aligned}$$

The method should also be applied to numbers such as $3.5\dot{1}\dot{2}$.

An alternative method is given in Topic 7.

- (iii) Determination of powers and roots, eg:

$$(2\frac{1}{3})^3; (0.8)^2; \sqrt{6\frac{1}{4}}; \sqrt{1.44}.$$

- (iv) Scientific notation and approximation. Interpretation of calculator output, rounding off to a given number of significant figures or decimal places, eg:

$$5\,230\,100 = 5\,230\,000 \text{ or } 5.230 \times 10^6 \text{ (correct to 4 significant figures),}$$

$$0.052\,073 = 0.0521 \text{ (correct to 4 decimal places)} \\ = 0.052\,07 \text{ (correct to 4 significant figures).}$$

In order to give an answer to a series of computations to a given accuracy, at least one additional figure must be used at each step of the computation and in obtaining the final answer, before rounding off.

- (v) Evaluation of expressions involving combinations of parentheses, powers, roots and the four operations, eg:

evaluate $\sqrt{(5^2 + 7^2)}$, correct to two decimal places;

find the exact value of $\frac{\frac{2}{5} + \frac{2}{3}}{1 - \frac{4}{15}}$.

- (vi) Quadratic surds — the four operations, with division done by rationalising the denominator, eg:

show that $\frac{4}{2 + \sqrt{5}} - \frac{1}{9 - 4\sqrt{5}}$ is rational.

Surdic equations and square roots of binomial surds are not included in this syllabus.

- 1.2 Inequalities should be reviewed, especially the effect of multiplication and division by negative numbers.

The absolute value $|a|$ equals a for $a \geq 0$, and $-a$ for $a < 0$.

The result $|a| = |a| \cdot |b|$ is important.

The result (the ‘triangle inequality’) $|a + b| \leq |a| + |b|$ should be derived.

The geometric interpretation of $|x|$ as the distance of x from the origin, and more generally, of $|x - y|$ as the distance between x and y (on the number line).

Simple graphs involving absolute values (see Topic 4.2).

- 1.3 Attention should be given to the following matters.

- (i) **Simplification** by removal of grouping symbols and collecting like terms, eg:

$$-5x - 3(2x + 1); 4(x^2 + 5x - 7) - 3(2x^2 - 7x + 1).$$

The addition, subtraction and multiplication of algebraic expressions, eg:

remove the parentheses from $(2x + 3)(x^2 + 5x + 2)$;
 subtract $5x + 2y - 3$ from $x - 7y + 9$.

(ii) **Substitution**

Evaluation of expressions involving the four operations, powers and roots. Numbers substituted may be integers, fractions, decimals or surds, eg:

find the exact value of $t^4 - t^2 + 1$ when $t = 2\sqrt{3}$;

find the exact value of

$$\frac{A^4 C}{B^4} \text{ where } A = \left(\frac{2}{3}\right)^2, B = \left(\frac{4}{3}\right)^4, C = \left(\frac{8}{3}\right)^7.$$

Problems involving substitution of numerical values into common formulae should be practised, eg:

given that $V = \pi r^2 h$, find the exact value
 of h , when $V = 10$, $r = 2$;

find t given that $s = ut + \frac{1}{2}at^2$, $a = 4$,
 $u = -4$, and $s = 6$.

(iii) **Factorisation**

Common factor, eg: $5x^2 - 10x = 5x(x - 2)$.

Difference of two squares, eg:

$$16x^2 - 1 = (4x + 1)(4x - 1).$$

Trinomials, eg: $t^2 - 4t + 4 = (t - 2)^2$;

$$3x^2 + 4x - 7 = (3x + 7)(x - 1).$$

Grouping of terms to involve the other types of factorisation, eg:

$$\begin{aligned} ax + ay - cx - cy &= a(x + y) - c(x + y) \\ &= (x + y)(a - c) \end{aligned}$$

$$\begin{aligned} x^2 - y^2 + 2x - 2y &= (x - y)(x + y) + 2(x - y) \\ &= (x - y)(x + y + 2). \end{aligned}$$

The sum and difference of two cubes, eg:

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4).$$

(iv) **Algebraic Fractions**

Reduction, eg:

$$\frac{5a - b}{2b - 10a}; \frac{x^2 - 5x + 6}{x - 2}.$$

Multiplication and division, eg:

$$\frac{3}{a - 2} \div \frac{a + 3}{a^2 - 4}.$$

Addition and subtraction, eg:

$$\frac{2m - n}{3} - \frac{m - 3n}{6}; \frac{2}{x} - \frac{3}{x(x + 2)}.$$

1.4 The following are included in this section.

(i) **Linear equations**, such as:

$$5t + 3 = 2(1 - t); \frac{3x + 4}{x} = 2;$$

$$\frac{3x - 1}{5x + 1} = \frac{3x - 2}{5x + 2}.$$

(ii) **Linear inequalities**, their solution and description on a number line, including problems involving absolute values, but not with the unknown in a denominator, eg:

find the values of x for which:

$$(a) 3x + 4 > 2\frac{1}{2}; (b) 3 - 2x \leq -1; (c) |x - 1| < 2.$$

(iii) **Quadratic equations**, including solution by factorisation and by formula, eg:

$$5x^2 - 11x + 2 = 0; 8t^2 = 1 - 10t.$$

$$y^2 = 6y; (v - 2)^2 = 16.$$

(iv) **Simultaneous equations**, only to the extent required by later topics. Students should always check the results by direct substitution in the original equations.

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(v) **Other inequalities**. 3 Unit students will be expected to be able to solve inequalities such as:

$$\frac{x^2 - 1}{x} > 0; \frac{2t + 1}{t - 2} > 1.$$